

Franck-Condon principle for heavy-quark hadron decays

Felipe Llanes-Estrada^{*}, Steve Cotanch[†], Ignacio General^{**} and Ping Wang[‡]

^{*}*Depto. Física Teórica I, Universidad Complutense de Madrid, 28040 Madrid, Spain*

[†]*Department of Physics, North Carolina State University, Raleigh NC 27695, USA*

^{**}*Bayer School of Natural and Environmental Sciences, Duquesne University, Pittsburgh, PA 15282, USA*

[‡]*Jefferson Laboratory, 12000 Jefferson Ave., Newport News, VA 23606, USA*

Abstract. The Franck-Condon principle governing molecular electronic transitions is utilized to study heavy-quark hadron decays. This provides a direct assessment of the wavefunction of the parent hadron if the momentum distribution of the open-flavor decay products is measured. Model-independent results include an experimental distinction between quarkonium and exotica (hybrids, tetraquarks...), an off-plane correlator signature for tetraquarks and a direct probe of the sea quark orbital wavefunction relevant in the discussion of 3S_1 or 3P_0 decay mechanisms.

Keywords: Franck-Condon, QCD exotica, quark momentum distribution, heavy-quark hadron decay.

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THE FRANCK-CONDON PRINCIPLE

Fluorescence spectra were first explained in 1925 by James Franck and Edward Condon on the basis of what is now known as the Franck-Condon [FC] principle. Recognizing the slow nuclear degrees of freedom, they asserted that for any molecular electronic transition (absorption or emission) there is no appreciable change in the internuclear coordinate separation or momentum (the scale separation between the electron mass and the nuclear mass being at least 10^{-3}). We propose that this principle can also be used to experimentally obtain the momentum distribution in heavy-quark systems. Our conjecture is that *the heavy-quark momentum distribution in the decaying hadron coincides with the momentum distribution of the decay-product hadrons each carrying a heavy quark*. That is, up to corrections of order Λ_{QCD}/M_Q (alternatively, α_s for ground state heavy quarkonium), and if E/p conservation permits, a measurement of the decay hadrons yields information about the parent hadron's wavefunction, thus allowing useful tests for exoticness, structure of the Fermi sea and other applications.

QUARKONIUM VERSUS HYBRID SIGNATURE

With the $\psi(4260)$, $\psi(4320)$ and $\psi(4620)$ recently discovered at B -factories and the $\psi(4040)$, $\psi(4160)$ and $\psi(4400)$ observed in the total cross section $e^-e^+ \rightarrow$ hadrons, there is now a clear excess of expected states in the charmonium spectrum. This overpopulation is explained in many-body approaches [1, 2, 3, 4] which predict several $c\bar{c}g$

hybrid mesons in addition to the $c\bar{c}$ spectrum, where four additional states are predicted in that The challenge for theorists is to identify and distinguish such states. To this end we propose using the model-independent FC principle to extract the parton probability distribution from experiment.

Consider the decay of the excited charmonium state, $\psi(4S)$ to $D\bar{D}\pi$. We focus on a three-body decay since in a two-body decay momentum conservation precludes using the FC principle. The momentum distribution of the c quarks is plotted in the first graph of Fig. 1. Invoking the FC conjecture, the momentum distribution of the decay D mesons is given by the second plot, now versus $|\mathbf{p}_D - \mathbf{p}_{\bar{D}}|$, in Fig. 1 which incorporates phase space and recoil effects from a finite quark mass. Note that after accounting for momentum smearing of order 150-200 MeV, the 3 Sturm-Liouville nodes, which must be preserved, are clearly depicted as dips in this spectrum.

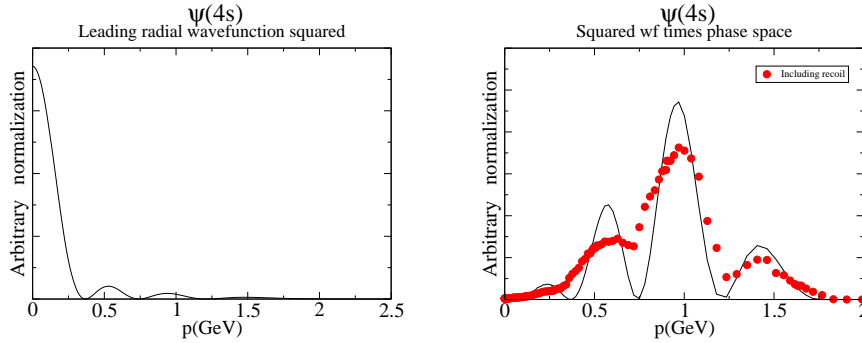


FIGURE 1. Left: the 4S quarkonium wavefunction in the Coulomb gauge model [1]. Right: the momentum distribution for the $D\bar{D}$ mesons in a three-body $D\bar{D}\pi$ decay, using the Franck-Condon principle for $c\bar{c} \rightarrow D\bar{D}$ (solid line) and momentum smearing (red dots). Note the preservation of wavefunction nodes.

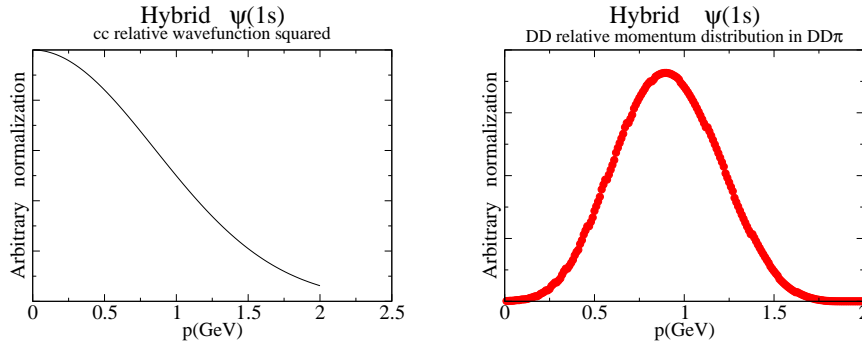


FIGURE 2. As in Fig. 1 for the ground state $c\bar{c}g$ hybrid meson with comparable mass in the 4.2-4.5 GeV region. Note the wavefunction is nodeless and the final relative momentum distribution of the $D\bar{D}$ pair has a simple bell-shape with no shoulders.

These distinctive dips are to be contrasted with the predicted [1, 2] bell-shaped momentum distribution in Fig. 2 that corresponds to a ground state $c\bar{c}g$ hybrid meson with comparable mass. The gluonic excitation is responsible for the additional energy that places this state high in the charmonium spectrum. The relative $c\bar{c}$ wavefunction in the hybrid state is nodeless and the corresponding decay spectrum is much simpler than in the case of an excited $c\bar{c}$. Hence by measuring the relative momentum of the D mesons

in the $D\bar{D}\pi$ subsystem, a model-independent way of distinguishing between a hybrid and quarkonium appears possible. Again note that $D\bar{D}$ decay alone is not useful since the two mesons momentum distribution is a Dirac delta function, which invalidates applying the FC principle. Related, $D\bar{D}^*$ decays also contribute to this three-body channel from a two-body decay which again avoids the FC constraint and need to be excluded with a kinematic cut in the Dalitz plot for $D\bar{D}\pi$.

Although the Belle collaboration has reported $D\bar{D}K$ spectra [5] stemming from weak B decays, this cannot be used for our studying since recoil corrections are larger for a kaon. We therefore await $D\bar{D}\pi$ data. Fortunately, the $b\bar{b}$ system provides an even more favorable application of the FC principle and the Belle collaboration has collected 20 fb^{-1} of integrated luminosity for the $\Upsilon(5S)$. This state, at 10860 MeV , has sufficient phase space to decay to $B\bar{B}\pi$ and can therefore be used to repeat the above analysis. This state's position is quite well reproduced by $b\bar{b}$ bottomonium models, and therefore its nature as (largely) a bottomonium excitation has not been questioned. The FC principle can be considered validated if the spectra of $B\bar{B}$ relative momenta in the $B\bar{B}\pi$ center of mass frame has visible shoulders.

OFF-PLANE CORRELATOR: TETRAQUARK SIGNATURE

From several studies [2, 3, 4] one can conclude that ground state tetraquarks and hybrids in the charmonium region can coexist with similar mass, and therefore the Sturm-Liouville nodal difference will not be useful. However, one can exploit the fact that a three-body hybrid meson has a planar structure with two independent momenta. The same structure emerges in the flux tube model, where the flux tube mode carries one unit of transverse excitation. In contrast, tetraquark systems have three independent momenta with one out of plane even in the center of momentum frame. Applying the Franck-Condon principle we deduce that a measurement of the off-planarity of the four-meson decay channel of the parent hadron measures the off-planarity of the latter in terms of its intrinsic wavefunction. Therefore one has to look for four-meson decay channels such as $D\bar{D}K\bar{K}$, and study their off-planarity in the center of momentum frame.

A measure of the deviation from planarity of a four-meson system has been presented in Ref. [3] where the off-plane correlator

$$\Pi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = \frac{((\mathbf{p}_1 \times \mathbf{p}_2) \cdot \mathbf{p}_3)^2}{\sqrt{|\mathbf{p}_1 \times \mathbf{p}_2| |\mathbf{p}_2 \times \mathbf{p}_3| |\mathbf{p}_1 \times \mathbf{p}_3| |\mathbf{p}_1 \times \mathbf{p}_4| |\mathbf{p}_2 \times \mathbf{p}_4| |\mathbf{p}_3 \times \mathbf{p}_4|}}$$

was introduced. The \mathbf{p}_i form a parallelepiped whose volume is zero if any three lie on the same plane. Therefore the volume $|\mathbf{p}_3 \cdot (\mathbf{p}_1 \times \mathbf{p}_2)|$ measures the deviation from coplanarity. However, this volume is also proportional to the length of the sides, i.e. depends on the phase space for the decay of the parent hadron and not only on its internal structure. The normalized correlator is a dimensionless, pure number, and therefore independent of the phase-space. It is also invariant under permutations and is zero if the Franck-Condon principle exactly holds for the decay of a meson with a two or three-body internal wavefunction. The maximum value of $\Pi \simeq 0.707$ seems to be attainable by a symmetric tetrahedral configuration of all 4 momenta, as we have checked by

Montecarlo simulations. A typical high value for this correlator is 0.59, corresponding to three of the mesons having equal momentum at right angles (along the edges of a cube), with the fourth momentum $\mathbf{p}_4 = -\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3$ longer by a factor $\sqrt{3}$.

As an example from current meson studies, we have analyzed the decay of the newly found $Z(4430)$ meson, observed by the Belle collaboration in the $\psi'\pi$ spectrum for the decay $B \rightarrow KZ \rightarrow K\psi'\pi$. The flavor quantum numbers of the Z meson are those of a π or ρ , namely $u\bar{d}$ for the Z^+ . However, its narrow width (40-50 MeV) for such a heavy meson, as well as its discovery decay mode, suggest that it has a significant component that is hidden-exotic, $u\bar{d}c\bar{c}$. Therefore having sufficient phase-space, it may decay $Z \rightarrow D\bar{D}\pi\pi$. Although in this example not all mesons have heavy quarks necessary for a rigorous application of the FC principle, it is still worthwhile to examine the distribution presented in Fig. 3.

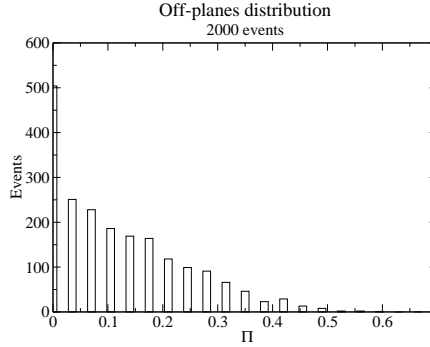


FIGURE 3. Distribution for a sample of 2000 four-meson events. The Montecarlo sample has been generated using a tetraquark model wave-function subject to the Franck-Condon constraint.

The calculation in the figure is the distribution of the off-plane correlator for a random sample of 2000 events, taking the four three-momenta of the quarks (that coincide exactly with the four meson-momenta only if the Franck-Condon principle exactly holds). The quark momenta have been generated with a standard pseudorandom number generator and distributed according to a model Gaussian wavefunction for the tetraquark, $\psi(p_1, p_2, p_3) = \mathcal{N}(p_I/\alpha_I) \exp(-\frac{p_A^2}{\alpha_A^2} - \frac{p_B^2}{\alpha_B^2} - \frac{p_I^2}{\alpha_I^2})$, with $\mathbf{p}_A = \mathbf{p}_1 - \mathbf{p}_2$, $\mathbf{p}_B = \mathbf{p}_3 - \mathbf{p}_4$ the cluster relative momenta and $\mathbf{p}_I = \mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4$ the intercluster momenta. The parameters are $\alpha_A = \alpha_B = 1.35 \text{ GeV}$, $\alpha_I = 0.6 \text{ GeV}$. See Ref. [3] for a complete discussion of the variational tetraquark wavefunction. As can be seen, the off-plane correlator is peaked at low values but is well-populated in the region 0.2-0.4 and even approaches the value found for the cubic configuration, 0.59. Again, even though the FC principle does not rigorously hold in this example, we submit that the non-zero correlator value is a representative result for tetraquark structure, quite distinct from the expected value of zero for a hybrid wavefunction.

It is interesting that a paper at this workshop by P. Bicudo and M. Cardoso also proposes that the $Z(4430)$ is a D^*D_1 molecule-type tetraquark decaying to $\psi'\pi^+$ and that the two D mesons have a node in their relative wavefunction. They obtain widths for this meson of order 0.2 MeV for the channel $J/\psi\pi^+$ and 4.6 MeV for the observed $\psi'\pi^+$, through the application of Moshinsky-Ribeiro-Van Beveren wavefunction overlap

rules. However, this is also equivalent to the Franck-Condon principle in reverse, where the original tetraquark meson has a wavefunction similar to the ψ' rather than the J/ψ as revealed by the decay pattern.

SPIN OF SEA PAIRS: DISTINGUISHING DECAY MECHANISMS

An unanswered question in hadron physics is whether the spins of quark-antiquark pairs from the hadronic Fermi sea are correlated. While deep inelastic scattering data [6] reveals that the sea quarks are largely unpolarized, it has not been used to extract quark-antiquark correlations. This question is important to understand meson decays since the valence quark component of the parent and daughter hadrons can be different which would require a pair creation transition. In QCD, quarks couple to negative parity vector gluons suggesting the $q\bar{q}$ pair are in a 3S_1 s -wave. However, extensive quark model phenomenology [7] indicates a 3P_0 scalar wavefunction. We argue here that the Franck-Condon principle can resolve this point in a model-independent way by extracting the correlation of a $c\bar{c}$ vacuum pair in a heavy-quark hadron decay. Such analysis would require a pair of $D\bar{D}$ mesons, and this favors B decays, since the B meson is heavy enough to decay to two D mesons, and copious samples, of order 500 million, have been produced at the B -factories. The channel we recommend for the analysis is the semileptonic decay $B \rightarrow e\bar{\nu} D\bar{D} n\pi$ where the two D mesons are accompanied by an electron and n pions as depicted in Fig. 4.

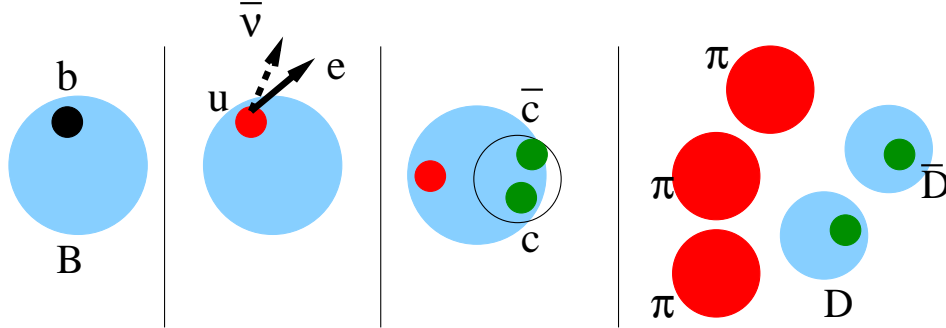


FIGURE 4. From left to right: a b quark in a B meson undergoes a CKM-suppressed weak decay. The excited hadron state decays by $c\bar{c}$ formation with relative orbital wavefunction to be determined. The final hadron decay products are two D mesons and an arbitrary number of pions. Whether the two D mesons are in a relative s or p -wave determines the corresponding angular wavefunction of the original $c\bar{c}$ pair by the Franck-Condon principle.

The semileptonic decays only account for about 10% of the total B samples and it is necessary to trigger with a fast electron to avoid charm quark counts from weak b decays, $b \rightarrow c\bar{c}s$ or $b \rightarrow c\bar{c}d$, that are a background to the strong charmed decays. Decays with one kaon or only one D meson are likewise to be discarded as a charmed quark could come from this weak decay. Unlike the previous application, $D^*\bar{D}$ and $D^*\bar{D}^*$ can now be used in place of $D\bar{D}$. The observable is the relative orbital wavefunction of the $D\bar{D}$ mesons, accompanied by any number of pions that balance energy-momentum. The D meson reconstruction efficiency is very low at B -factories, around 1%, and the B meson decays to D pairs also have a small branching fraction of 1% respect to similar light-

light decays [8]. We thus expect samples of order 10^{-5} of the total number of events. It is not necessary to reconstruct the B meson decay, so the b quark may be tagged by the other B meson in the $Y(4S) \rightarrow B\bar{B}$ reaction. The two D mesons however need to be fully reconstructed and their momentum measured accurately enough to perform an angular analysis to distinguish between a s or p wave relative wavefunction.

SUMMARY

We have shown that the Franck-Condon principle can be an effective tool to experimentally extract information about the wavefunction in heavy quark systems, especially for documenting exotic degrees of freedom. We believe further applications of this constraint in high statistics measurements will provide significant hadron structure insight.

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